

# The Improved Algorithm of the EMD Decomposition Based on Cubic Spline Interpolation

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## Abstract

It is critical to reduce the abnormal and complex jamming signals in the first order IMF component, because the decomposition result of the first order intrinsic mode function (IMF) in the empirical mode decomposition (EMD) method directly affects the subsequent decomposition effect, and also there are sampling restrictions for ultrasonic detectors. In this paper, we use a three-step optimization method to obtain the first order IMF. The first step is that the local extreme points of collected echo signals are fitted as the upper and lower envelopes by using the cubic spline interpolation method. The second step is to look for the curve of local mean of the echo signals on the basis of the upper and lower envelopes. The third step decomposes the curve of local mean by the EMD method. The simulation results show that our proposed method can effectively reduce jamming signals in the first order IMF and achieve the aim of improving the decomposition effect of the EMD method.

## Keywords

*Empirical Mode Decomposition; Cubic Spline Interpolation; Intrinsic Mode Function; The Curve of Local Mean*

## Introduction

In modern industry, metal and nonmetal bonding materials are widely used in the National Defense Industry because of its excellent performance of composite. The quality of composite bonding materials has a decisive impact on production usage. One of many methods to identify the internal quality of products is the ultrasonic test technology. The principle of ultrasonic test is based on the propagation characteristics of ultrasonic in specimens. Firstly, the ultrasonic probe produces ultrasonic and uses a certain way to make it into specimen, then the ultrasonic travels along a straight line in the specimen. In the meantime it interacts with specimen material and its weaknesses. This makes the direction of propagation and characters of the ultrasonic changed. The ultrasonic whose direction of propagation is changed is received and displayed on the oscilloscope by the ultrasonic detecting device. This is called the echo signals. Then the echo signals are processed and analyzed by appropriate methods. Finally, the internal defects of bonding materials are evaluated according to the characteristics of the received ultrasonic.

Echo signals detected by ultrasonic apparatus have non-stationary and nonlinear characteristics. In the face of such signals, they can be decomposed into a number of IMFs and a residual amount by using the EMD method. Every IMF contains the local features of signals in time domain and frequency domain, and the residual amount is expressed as variation trend of signals in time domain. In recent years, with the development of ultrasonic test technology, many new methods of signal processing are used for detecting structural damage. Since the Hilbert-Huang Transform (HHT) method based on the EMD has many advantages in dealing with nonlinear and non-stationary signals, this method has got the attention of researchers in the field of test engineering and got promotion application quickly. However, due to the EMD's own algorithm, the decomposition result of the first order IMF affects the subsequent decomposition effect. Therefore it is necessary to improve the decomposition result of the first order IMF. Taking into account that the EMD method is a data processing method based on the time domain, the accuracy of decomposition highly requires the data accuracy and smoothness. In this paper, an improved method based on the cubic spline interpolation is proposed, which represents a substantial improvement

over the original EMD. We use a three-step optimization method to obtain the first order IMF. The first step is that the local extreme points of collected echo signals are fitted as the upper and lower envelopes by using the cubic spline interpolation method. The second step is to look for the curve of local mean of the echo signals on the basis of the two envelopes. The third step decomposes the curve of local mean by the EMD method. The experimental analysis shows that the improved EMD method is very effective in the treatment of ultrasonic flaw detection.

### Principle of Empirical Mode Decomposition

Empirical mode decomposition method, as a new method of digital signal analysis, was proposed by N.E Huang in 1998, who was an academican of the National Aeronautics and Space in America. The EMD method decomposes signals according to the time scale characteristic of the data, which does not need to set any basis functions. Because of these characteristics, the EMD method can be applied to any type of signals in theory. In dealing with non-stationary and nonlinear signals, it has obvious advantages. The EMD is a "filtering method" to decompose a complex signal into a finite number of IMFs and a residual component. For a given signal, for example  $x(t)$ , the basic steps of the EMD method are as follows:

- (1) Identify all the local extreme points of signal  $x(t)$ ;
- (2) Connect all these local extreme points with the cubic spline interpolation as the upper envelope  $u(t)$  and lower envelope  $v(t)$ ;
- (3) Obtain the curve of local mean of the upper and lower envelopes  $m(t) = \frac{u(t) + v(t)}{2}$ ;
- (4) Calculate the deviation between  $x(t)$  and  $m(t)$ :  $h_1(t) = x(t) - m(t)$ ;
- (5) Treat  $h_1(t)$  as a new original signal and repeat the above steps as many times as required until the envelopes are symmetric with respect to zero mean under certain criteria, then obtain the first order IMF component, that is  $c_1(t) = h_1(t)$ ;
- (6) The original signal  $x(t)$  minus  $c_1(t)$  to continue repeating the above process until separate all IMFs, denoted by  $c_1(t), c_2(t), \dots, c_n(t)$ , and list the following formula:

$$x(t) = \sum_{i=1}^n c_i(t) + r_n(t)$$

where  $r_n(t)$  is the residue of signal  $x(t)$  and  $n$  is the total number of IMFs generated from the original signal  $x(t)$ .  $c_i(t)$  contains different frequency components of the original signal from high to low.

### Cubic Spline Interpolation

Because the local extreme points of the original signals were fitted as the upper and lower envelopes by using the cubic spline interpolation in the EMD method, we use the cubic spline interpolation method to smooth the original signal in order to alleviate the error of fitting signal envelopes. The curve can better characterize the extrema of the original signals and achieve the purpose of improving the accuracy of EMD algorithm.

The cubic spline interpolation is a smooth curve through a series of points. Contrasting to other methods, the advantage of the cubic spline interpolation method is to improve the smoothness of the piecewise linear interpolation function at nodes. For given different nodes  $x_0, x_1, x_2, \dots, x_n$ , the corresponding values are  $y_0, y_1, y_2, \dots, y_n$ . The cubic spline interpolation function  $S(x)$  in the range of  $a = x_0 < x_1 < x_2 < \dots < x_n = b$  should meet the following conditions:

- (1) It has the second derivative in the range of  $[a, b]$ ;
- (2)  $S(x_k) = y_k$  ( $k=0, 1, 2, \dots, n$ );
- (3)  $S(x)$  is a thrice polynomial in each sub interval  $[x_k, x_{k+1}]$  ( $k=0, 1, 2, \dots, n-1$ ).

## Simulation and Analysis

In this paper, we use MATLAB programming software to construct a kind of cubic spline fitting method, which can smooth the echo signals. The reason to smooth the signals is that the collected signals are limited by the instrument which has the limitations of the sampling effect, then the amplitude of echo signals mutates more frequently (as shown in Fig. 1). If we use EMD method to process such echo signals directly, the first order IMF in the decomposition will have a lot of abnormal and complex jamming signals, which will affect the subsequent decomposition effect. Here we take an echo signal for example which comes from a thin board with a viscosity of 30%.

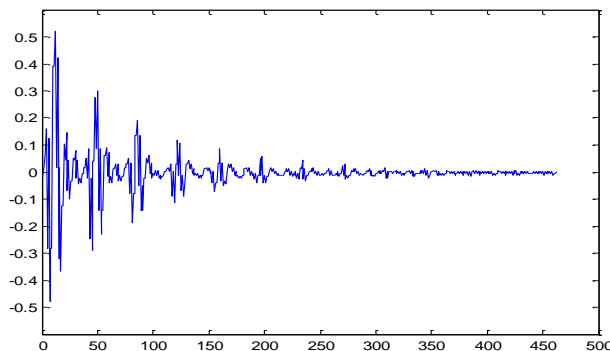


FIG. 1 THE ECHO SIGNAL WITH A VISCOSITY OF 30%

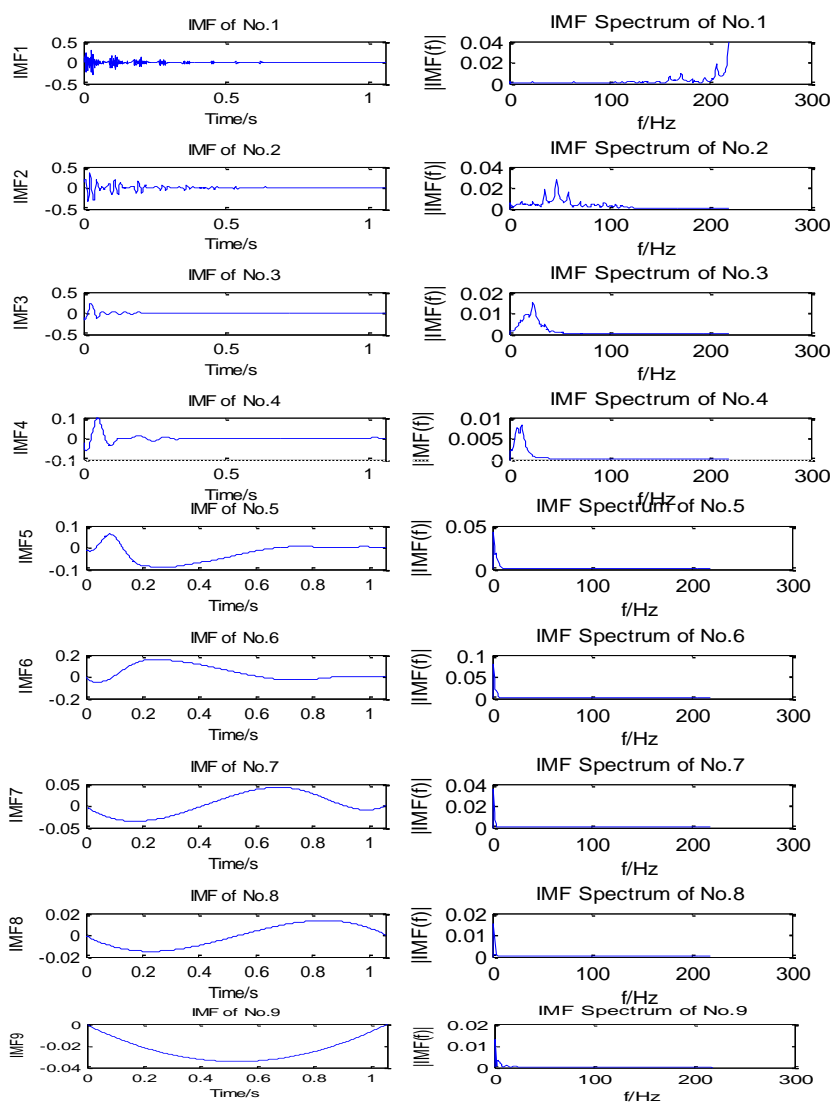


FIG. 2 EACH ORDER IMF COMPONENT AND ITS SPECTRUM

Fig. 1 shows the echo waveform, which is collected by ultrasonic detector and does not do any processing. It can be seen from the figure that, the echo signal works as simple harmonic oscillation and it is made up of multiple echoes, which are gradually weakened with the increase of the propagation distance. The vertical coordinate of the echo signal stands for the amplitude of the wave, and it also represents the intensity of the echo signal that is produced by the ultrasonic which is emitted into specimen. The horizontal coordinate of the echo signal stands for the sampling points, and also represents the sound path. After decomposing the untreated echo signal by EMD method, the IMF components and the spectrum of each IMF are shown in Fig. 2.

From Fig. 2 it can be seen that the frequency band of the first order IMF is wide, and there are many jamming signals in it. As the result of the first order IMF is the most influential to the following decomposition effect, and if we can reduce the error of the first order IMF, the decomposition accuracy of the whole data can be improved. So the local maximum points and local minimum points of the echo signal are fitted by cubic spline interpolation method, and upper envelope  $s_1(t)$  and lower envelope  $s_2(t)$  are came into being. Then we obtain the curve of local mean of the upper and lower envelopes:

$$x(t) = \frac{s_1(t) + s_2(t)}{2}$$

(as shown in Fig. 3, Fig. 4 shows the curve of local mean  $x(t)$ ). Finally  $x(t)$  is decomposed by EMD method, and the decomposition process is shown in Fig. 5.

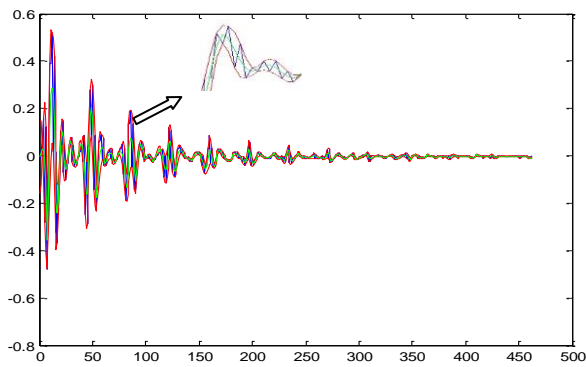


FIG. 3 SMOOTHING PROCESS OF THE ECHO SIGNAL

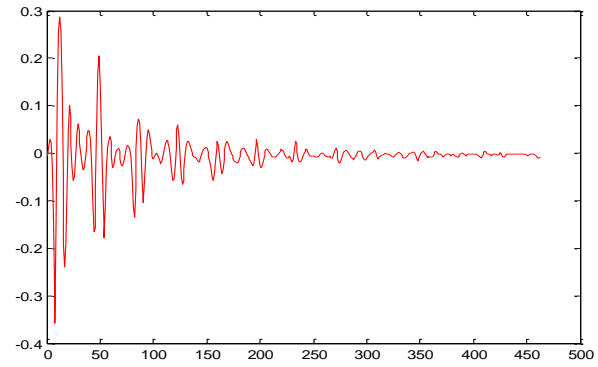


FIG. 4 THE CURVE OF LOCAL MEAN

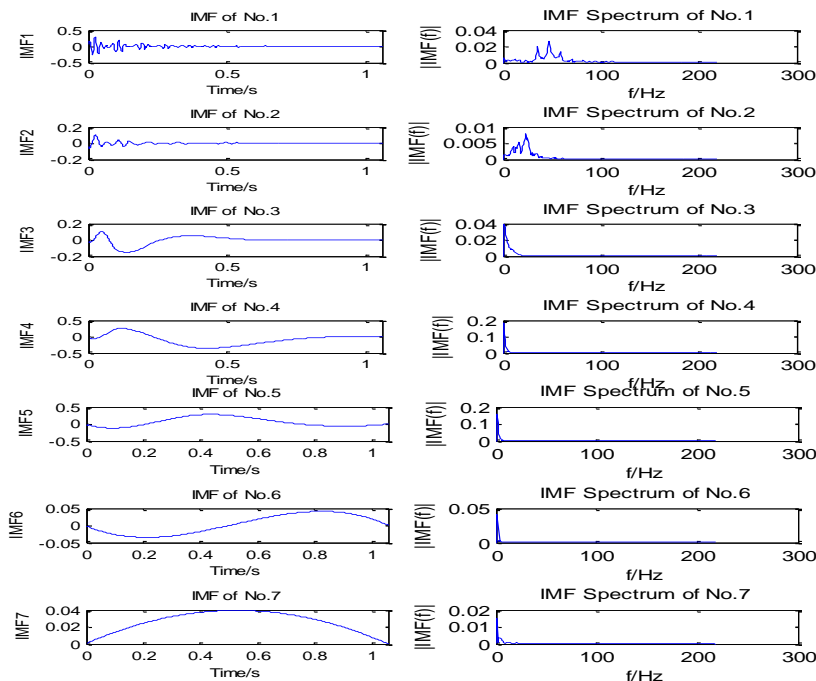


FIG. 5 EACH ORDER IMF COMPONENT AND THEIR SPECTRUMS AFTER SMOOTHING

As seen from Fig. 5, the frequency band of the first order IMF is much narrower than it is shown in Fig. 2. That is to say, the number of jamming signals in the first order IMF is reduced after smoothing the signal by cubic spline interpolation method. The subsequent decomposition effect is better than it before.

### Evaluating Indicator

In order to evaluate the accuracy of the IMF component between original EMD method and the new EMD method, in this paper, the correlation coefficient NC is chosen as the evaluating indicator. It is the degree of correlation between the IMF component and the corresponding signal. With this parameter the precision of IMF component is evaluated. Definition of the correlation coefficient NC is shown as (3-1):

$$NC = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^N (x_i - \bar{x})^2} \cdot \sqrt{\sum_{i=1}^N (y_i - \bar{y})^2}} = \frac{N \sum_{i=1}^N x_i y_i - \sum_{i=1}^N x_i \cdot \sum_{i=1}^N y_i}{\sqrt{N \sum_{i=1}^N x_i^2 - (\sum_{i=1}^N x_i)^2} \cdot \sqrt{N \sum_{i=1}^N y_i^2 - (\sum_{i=1}^N y_i)^2}} \quad (3-1)$$

The performance comparison between traditional EMD method and the new EMD method is shown in Table 1. From the table we can see that, the NC of the first order IMF component obtained by the new EMD method is higher than that of the first order IMF component which is obtained by the traditional EMD method. That is to say, the accuracy of the first order IMF component obtained by the new EMD method is higher than that obtained by the traditional EMD method. In addition, the NC of the second and third order IMFs obtained by the new EMD method is lower than the corresponding IMFs obtained by the traditional EMD method. And the NC of the other IMFs are higher than those of the same order IMFs obtained by the traditional EMD method. It is further explained that the improved new EMD method is very effective in the ultrasonic testing technology.

TABLE 1 COMPARISON OF PERFORMANCE EVALUATING INDICATOR OF DIFFERENT EMD METHODS

correlation coefficient EMD methods	NC (IMF1)	NC (IMF2)	NC (IMF3)	NC (IMF4)	NC (IMF5)	NC (IMF6)	NC (IMF7)	NC (IMF8)	NC (IMF9)
Traditional EMD	0.6159	0.6811	0.2382	0.0169	-0.0031	-0.0005	-0.0126	-0.0098	0.0103
Smoothing EMD	0.9203	0.2413	0.0465	-0.0213	0.0127	0.0334	0.0291		

### Conclusions

In ultrasonic test, the sampling effect of echo signals is limited by the instrument and other factors. The frequency band of the first order IMF obtained by EMD method is wide, and it comprises many jamming signals. In the traditional EMD method, the decomposition error of the first order IMF will directly affects the result of the following decomposition. In this paper, an improved method based on the cubic spline interpolation is proposed. We use a three-step optimization method to obtain the first order IMF. The first step is that the local extreme points of collected echo signals are fitted as the upper and lower envelopes by using the cubic spline interpolation method. The second step is to look for the curve of local mean of the echo signals on the basis of the two envelopes. The third step decomposes the curve of local mean by the EMD method. The experiment result shows that the optimization EMD method is significant for analyzing the breakdown signals containing complex and abnormal events.

### REFERENCES

- [1] Huang N E. The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Non-Stationary Time Series Analysis[J]. Proceedings of the Royal Society London A,1998, 454:903—995.
- [2] Zhaohua W, Huang N E. Ensemble Empirical Mode Decomposition: a noise assisted data analysis method[J]. Advances in Adaptive Data Analysis,2008,1(1): 1-41.
- [3] Aijun Hu, Jingjing Sun, XiangLin. Mode Mixing Problem In Empirical Mode Decomposition[J]. Journal of Vibration, Measurement & Diagnosis,2011,31(4):428-434.

- [4] Wenmao Yang. A Problem of Using The First or Second Derivative of Each Node or The Third Derivative of Each Line to Determine The Cubic Spline[J]. Journal of Wuhan University, Doi:10.14188 /j.1671-8836.1981.04.001.
- [5] Xiaoyong Xu, Taiyong Zhong. Structure of The Cubic Spline Interpolation Function and MATLAB Realization[J]. Automatic Measurement and Control, 2006,25(11):76-78.
- [6] Zhenggan Zhou, XiangShang, WeiDong. Ultrasonic Testing Technologies for Composites[J]. Aviation Manufacturing Technology,2009,8:70-73.
- [7] Chen H G, Yan Y J, Jiang J S. Vibration-Based Damage Detection in Composite Wing Box Structures by HHT[J]. Mechanical Systems and Signal Processing, 2007, 21:307-321.
- [8] Xiumin li, Weihua Jiang. Correlation Coefficient and Correlation Measure[J]. Mathematics In Practice and Theory,2006,36(12):188-192.
- [9] Ryuho Kataoka, Yoshizumi Miyoshi, Akira Morioka. Hilbert-Huang Transform of Geomagnetic Pulsations at Auroral Expansion onset[J].Journal of Geophysical Research, Doi:10.1029/2009JA014214